National
Qualifications
2016

2016 Mathematics of Mechanics

Advanced Higher

Finalised Marking Instructions

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General Marking Principles for Advanced Higher Mathematics of Mechanics

This information is provided to help you understand the general principles you must apply when marking candidate responses to questions in this Paper. These principles must be read in conjunction with the detailed marking instructions, which identify the key features required in candidate responses.

For each question the marking instructions are generally in two sections, namely Illustrative Scheme and Generic Scheme. The Illustrative Scheme covers methods which are commonly seen throughout the marking. The Generic Scheme indicates the rationale for which each mark is awarded. In general, markers should use the Illustrative Scheme and only use the Generic Scheme where a candidate has used a method not covered in the Illustrative Scheme.

(a) Marks for each candidate response must always be assigned in line with these General Marking Principles and the Detailed Marking Instructions for this assessment.

(b) Marking should always be positive. This means that, for each candidate response, marks are accumulated for the demonstration of relevant skills, knowledge and understanding: they are not deducted from a maximum on the basis of errors or omissions.

(c) If a specific candidate response does not seem to be covered by either the principles or detailed Marking Instructions, and you are uncertain how to assess it, you must seek guidance from your Team Leader.

(d) Credit must be assigned in accordance with the specific assessment guidelines.

(e) One mark is available for each •. There are no half marks.

(f) Working subsequent to an error must be **followed through**, with possible credit for the subsequent working, provided that the level of difficulty involved is approximately similar. Where, subsequent to an error, the working for a follow through mark has been eased, the follow through mark cannot be awarded.

(g) As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Unless specifically mentioned in the marking instructions, a correct answer with no working receives no credit.

(h) Candidates may use any mathematically correct method to answer questions except in cases where a particular method is specified or excluded.

(i) As a consequence of an error perceived to be trivial, casual or insignificant, eg $6 \times 6 = 12$ candidates lose the opportunity of gaining a mark. However, note the second example in comment **(j)**.

(j) Where a transcription error (paper to script or within script) occurs, the candidate should normally lose the opportunity to be awarded the next process mark, eg

$$x^2+5x+7=9x+4$$

This is a transcription error and so the mark is not awarded.

$$x-4x+3=0$$

Eased as no longer a solution of a quadratic equation so mark is not awarded.

$$x=1$$

$$x^2+5x+7=9x+4$$

$$x-4x+3=0$$

Exceptionally this error is not treated as a transcription error as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.

$$(x-3)(x-1)=0$$

$$x=1 \text{ or } 3$$

(k) **Horizontal/vertical marking**
Where a question results in two pairs of solutions, this technique should be applied, but only if indicated in the detailed marking instructions for the question.

Example:

	•5	•6
•5	$x = 2$	$x = -4$
•6	$y = 5$	$y = -7$

Horizontal: •5 $x = 2$ and $x = -4$
•6 $y = 5$ and $y = -7$

Vertical: •5 $x = 2$ and $y = 5$
•6 $x = -4$ and $y = -7$

Markers should choose whichever method benefits the candidate, but **not** a combination of both.

(l) In final answers, unless specifically mentioned in the detailed marking instructions, numerical values should be simplified as far as possible, eg:

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$

$\frac{43}{1}$ must be simplified to 43

$\frac{15}{0\cdot3}$ must be simplified to 50

$\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 100 must be known.

(m) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(n) Unless specifically mentioned in the marking instructions, the following should not be penalised:

- Working subsequent to a correct answer
- Correct working in the wrong part of a question
- Legitimate variations in numerical answers/algebraic expressions, eg angles in degrees rounded to nearest degree
- Omission of units
- Bad form (bad form only becomes bad form if subsequent working is correct), eg $(x^3+2x^2+3x+2)(2x+1)$ written as $(x^3+2x^2+3x+2)\times 2x+1$

  $2x^4+4x^3+6x^2+4x+x^3+2x^2+3x+2$ written as $2x^4+5x^3+8x^2+7x+2$ gains full credit

- Repeated error within a question, but not between questions or papers

(o) In any 'Show that…' question, where the candidate has to arrive at a required result, the last mark of that part is not available as a follow-through from a previous error unless specified in the detailed marking instructions.

(p) All working should be carefully checked, even where a fundamental misunderstanding is apparent early in the candidate's response. Marks may still be available later in the question so reference must be made continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that the candidate has gained all the available marks.

(q) Scored-out working which has not been replaced should be marked where still legible. However, if the scored out working has been replaced, only the work which has not been scored out should be marked.

(r) Where a candidate has made multiple attempts using the same strategy and not identified their final answer, mark all attempts and award the lowest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

(s) Any rounded answer should be accurate to three significant figures (or one decimal place for angles given in degrees) unless otherwise stated. If an answer differs due to rounding or prior rounding the candidate may be penalised. Only penalise one mark in any question.

Detailed Marking Instructions for each question

Question			Generic Scheme	Illustrative Scheme	Max Mark
1.			•^{1} calculate impulse	•^{1} $F \times t = -180 \times 1\cdot5 = -270\,\text{Ns}$	**3**
			•^{2} equate impulse to change in momentum	•^{2} $-270 = mv - mu = m(v-u)$	
			•^{3} calculate change in velocity and final velocity	•^{3} $v = 12 - \frac{270}{70} = 12 - 3\frac{6}{7} = 8\frac{1}{7}(\text{m s}^{-1})(8\cdot14)$	
Notes: 1. Direction of motion must be implied in either •^{1} or •^{2}					
Commonly Observed Responses:					
			Alternative solution		
			•^{1} Use $F = ma$ to find acceleration	•^{1} $-180 = 70a \Rightarrow a = -\frac{18}{7}\text{m s}^{-2}$	
			•^{2} Use equations of motion with substitution	•^{2} $v = u + at$ with indication of direction of motion	
			•^{3} Correct value for velocity	•^{3} $v = 12 - \frac{18}{7}(1\cdot5) = 12 - 3\frac{6}{7}\text{m s}^{-1}$ $= 8\frac{1}{7}\text{ms}^{-1}\left(8\cdot14\text{ms}^{-1}\right)$	
Notes:					
Commonly Observed Responses:					
2.			•^{1} Resolve in x direction	•^{1} $P\cos30^\circ - Q\cos60^\circ - 80\cos60^\circ = 0$	**4**
			•^{2} Resolve in y direction	•^{2} $Q\sin60^\circ + P\sin30^\circ - 80\sin60^\circ - 64 = 0$	
			•^{3} Algebraic manipulation to find value of P	•^{3} $P = 40\sqrt{3} + 32 \approx 101\text{N}$	
			•^{4} Find value of Q	•^{4} $Q = 40 + 32\sqrt{3} \approx 95\cdot4\text{N}$	
Notes:					
Commonly Observed Responses:					

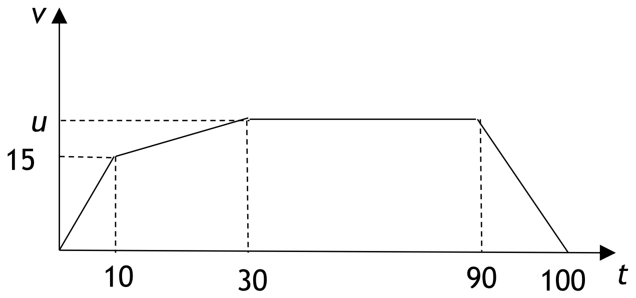
Question			Generic Scheme	Illustrative Scheme	Max Mark
3.			•^{1} calculate the displacement	•^{1} $\mathbf{d}=\begin{pmatrix}6\\4\end{pmatrix}-\begin{pmatrix}-3\\1\end{pmatrix}=\begin{pmatrix}9\\3\end{pmatrix}$	**3**
			•^{2} substitute into formula for work done	•^{2} work done $=\mathbf{F}\bullet\mathbf{d}$ $=\begin{pmatrix}2\\3\end{pmatrix}\bullet\begin{pmatrix}9\\3\end{pmatrix}$	
			•^{3} calculate the work done	•^{3} $=18+9$ $=27\,\mathrm{J}$	
Notes:					
Commonly Observed Responses: 1. Candidates did not know how to use scalar product and gave answer as a vector $\begin{pmatrix}18\\9\end{pmatrix}$					
4.			$y=2x-e$		**3**
			•^{1} Show understanding of use of product rule	•^{1} $\frac{dy}{dx}=1\left(\frac{d}{dx}\left(\frac{1}{x}\right)\right)+x\frac{d}{dx}(\ln x)$	
			•^{2} Correct differentiation	•^{2} $\frac{dy}{dx}=1+\ln x$	
			•^{3} Find equation of tangent	•^{3} when $x=e$ $y=e\ln e=e$ and $\mathrm{m}=2$ $y-e=2(x-e)$	
Notes: 1. Simplification not required for •^{1} or •^{3}					
Commonly Observed Responses:					

Question			Generic Scheme	Illustrative Scheme	Max Mark
5.			$\bullet^{1}$ Substitute values into SHM eqs $v^2=\omega^2\left(a^2-x^2\right)$	$\bullet^{1}$ $4=\omega^2\left(a^2-0\cdot005^2\right)$ or $1=\omega^2\left(a^2-0\cdot007^2\right)$	5
			$\bullet^{2}$ Obtain second equation and equate expressions for ω^2	$\bullet^{2}$ second equation and $\omega^2=\dfrac{4}{a^2-0\cdot005^2}=\dfrac{1}{a^2-0\cdot007^2}$	
			$\bullet^{3}$ Find value of *a*	$\bullet^{3}$ $a^2=5\cdot7\times10^{-5}$ $a=7\cdot55\times10^{-3}\ \text{m}$	
			$\bullet^{4}$ Find value of ω	$\bullet^{4}$ $\omega^2=\dfrac{4}{5\cdot7\times10^{-5}-0\cdot005^2}$ $\omega=353\cdot6\,\text{rad}\,\text{s}^{-1}$	
			$\bullet^{5}$ Calculate frequency	$\bullet^{5}$ $\dfrac{353\cdot6}{2\pi}=56\cdot3$ oscillations per second	

Notes:

Commonly Observed Responses:

1. 5mm and 7mm not converted to metres

Question			Generic Scheme	Illustrative Scheme	Max Mark
6.	(a)		•^{1} Correct shape of graph •^{2} All annotations correct  		**2**
Notes:					
Commonly Observed Responses:					
	(b)	(i)	•^{1} Find area under graph and equate •^{2} Calculate value of *u*	•^{1} $\frac{1}{2}(10\times15)+\frac{1}{2}(u+15)\times20$ $+60u+\frac{1}{2}(10\times u)=1725$ •^{2} $u=20$	**2**
Notes:					
Commonly Observed Responses:					
		(ii)	•^{1} One assumption stated	•^{1} Path of aircraft remains in one plane (2 dimensions) Each section is in vertical plane (no wobble)	**1**
Notes:					
Commonly Observed Responses: 1. The assumption given had to be about *the **PATH*** of the aircraft					

Question			Generic Scheme	Illustrative Scheme	Max Mark
7.			•¹ Find time for maximum speed	•¹ max speed when $a=0$ $4-\sqrt{t}=0$ $t=16\text{secs}$	**4**
			•² Integrate to give expression of velocity with evidence of $C=0$ if indefinite integration used.	•² $v=4t-\frac{2}{3}t^{\frac{3}{2}}+C$ at $t=0, v=0 \Rightarrow C=0$, $v=4t-\frac{2}{3}t^{\frac{3}{2}}$	
			•³ Calculate velocity after 16 seconds	•³ $v_{16}=4(16)-\frac{2}{3}(16)^{\frac{3}{2}}=\frac{64}{3}\text{ms}^{-1}$	
			•⁴ Calculate increase in kinetic energy	•⁴ change in energy $=\frac{1}{2}\times 9\times\left(\frac{64}{3}\right)^2-0$ $=2048\text{J}\ (2050\text{J})$	
			Alternative solution for •³ and 4		
			•³ obtain an expression for the work done as an integral	•³ $F=ma$ $F=36-9t^{\frac{1}{2}}$ $W=\int F.vdt$ $W=\int_0^{16}(36-9t^{\frac{1}{2}})(4t-\frac{2}{3}t^{\frac{3}{2}})dt$	
			•⁴ integrate and solve to obtain the work done	•⁴ $W=\int_0^{16}(144t-60t^{\frac{3}{2}}+6t^2)dt$ $=\left[72t^2-24t^{\frac{5}{2}}+2t^3\right]_0^{16}$ $=2048\text{J}\ (2050\text{J})$	

Notes:

Commonly Observed Responses:

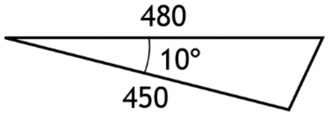
1. •¹ Common error $4-\sqrt{t}=0 \Rightarrow t=2$

Question			Generic Scheme	Illustrative Scheme	Max Mark
8.	(a)		•¹ Show equivalence of denominators	•¹ $(x+1)(x+3)(x-2)=x^3+2x^2-5x-6$	**3**
			•² Divide polynomials to obtain a remainder	•² $\begin{array}{r} 3 \\ x^3+2x^2-5x-6\big)\overline{3x^3+8x^2-11} \end{array}$ with any remainder stated	
			•³ Complete proof	•³ $3+\frac{2x^2+15x+7}{x^3+2x^2-5x-6}$ or $3+\frac{2x^2+15x+7}{(x+1)(x+3)(x-2)}$	
Notes:					
Commonly Observed Responses:					
	(b)		•¹ Correct form of partial fractions	•¹ $\frac{A}{(x+1)}+\frac{B}{(x+3)}+\frac{C}{(x-2)}$	**4**
			•² Form equation	•² $2x^2+15x+7=A(x+3)(x-2)$ $+B(x+1)(x-2)+C(x+1)(x+3)$	
			•³ Any two constant values	•³ $x=-1\Rightarrow A=1$ and/or $x=-3\Rightarrow B=-2$ and/or $x=2\Rightarrow C=3$	
			•⁴ Remaining value and express in correct form	•⁴ $3+\frac{1}{x+1}-\frac{2}{x+3}+\frac{3}{x-2}$	
Notes:					
Commonly Observed Responses:					

Question			Generic Scheme	Illustrative Scheme	Max Mark
9.	(a)		•¹ Horizontal forces: $F = ma$ with substitution	•¹ $R\sin 32^{\circ} + 0\cdot3R\cos 32^{\circ} = \frac{mv^2}{30}$	**6**
			•² Vertical forces in equilibrium with substitution	•² $R\cos 32^{\circ} - 0\cdot3R\sin 32^{\circ} = mg$	
			•³ Algebraic manipulations	•³ $\frac{\sin 32^{\circ} + 0\cdot3\cos 32^{\circ}}{\cos 32^{\circ} - 0\cdot3\sin 32^{\circ}} = \frac{v^2}{30g}$	
			•⁴ Value of v	•⁴ $v = 18\cdot3\text{ms}^{-1}$	
			•⁵ Calculate time for 1 lap	•⁵ $C = \pi d = \pi \times 60 = 188\cdot5$m $t = \frac{d}{v} = \frac{188\cdot5}{18\cdot3} = 10\cdot3$ seconds	
			•⁶ Calculate number of laps and round down	•⁶ $300 \div 10\cdot3 = 29\cdot1 \Rightarrow 29$ laps	
			Alternative solution to •⁵ and •⁶		
			•⁵ Find distance travelled in 5 minutes	•⁵ $18\cdot3 \times 300 = 5490\,\text{m}$	**6**
			•⁶ Calculate number of laps and round down	•⁶ $5490 \div 60\pi = 29\cdot1 \Rightarrow 29\,\text{laps}$	
Notes:					
Commonly Observed Responses: 1. Friction was ignored 2. Many candidates do not understand that motion here is horizontal and choose to treat this as motion on a slope by considering equilibrium perpendicular to the slope and then use acceleration along the slope.					
	(b)		•¹ Assumption stated	•¹ Track of negligible width **or** cyclist remains 30 metres from the centre of the horizontal circle.	**1**
Notes:					
Commonly Observed Responses:					

Question			Generic Scheme	Illustrative Scheme	Max Mark
10.	(a)		•¹ Differentiate x and y with respect to time •² Find $\frac{dy}{dx}$	•¹ $\frac{dx}{dt}=4$ and $\frac{dy}{dt}=2-10t$ •² $\frac{dy}{dx}=\frac{2-10t}{4}\left(=\frac{1-5t}{2}\right)$	**2**
Notes:					
Commonly Observed Responses:					
	(b)	(i)	•¹ Substitute t = 0 to give the initial velocities horizontally and vertically •² Find angle	•¹ $t=0:\frac{dy}{dx}=\frac{2}{4}=0\cdot5$ •² $\tan^{-1}(0\cdot5)=26\cdot6^{0}$ or $0\cdot464$	**2**
Notes:					
Commonly Observed Responses:					
		(ii)	•¹ Condition for angle of 45^{0} below horizontal •² Solve for time	•¹ $\frac{dy}{dx}=-1$ •² $t=\frac{6}{10}=0\cdot6$ seconds	**2**
Notes					
Commonly Observed Responses: **1.** In b(ii) $\frac{dy}{dx}=1$ gives an answer of $t=-0\cdot2$ seconds which cannot be explained.					

Question			Generic Scheme	Illustrative Scheme	Max Mark
11.			•¹ Find the area under the curve •² State the correct integral for $A\bar{x}$ with substitution •³ State the correct integral for $A\bar{y}$ with substitution • ⁴ Find the centre of mass	• ¹ Shaded area = $\int_0^4 x^3dx = 64$ sq units •² $A\bar{x} = \int_0^4 xydx = \int_0^4 x^4dx = 204\cdot8$ $\bar{x} = \frac{204\cdot8}{64} = 3\cdot2$ •³ $A\bar{y} = \int_0^4 \frac{1}{2}y^2dx = \int_0^4 \frac{x^6}{2}dx = 1170\cdot3$ $\bar{y} = \frac{1170\cdot3}{64} = 18\cdot3$ •⁴ Centre of Mass: $(3\cdot2, 18\cdot3)\left[\frac{16}{5}, \frac{128}{7}\right]$	**4**
Notes:					
Commonly Observed Responses: 1. Formula for $A\bar{y}$ not known.					
			Alternative solution for •³ and • ⁴		
			•3 Use method as A$\bar{x}$ but Replacing x and y •4 Find the centre of mass	$A\bar{y} = \int_0^{64} y(4-x)dy = \int_0^{64} y(4-y)^{\frac{1}{3}}dy$ $= \int_0^{64} (4y - y^{\frac{4}{3}})dy = \frac{8192}{7} \Rightarrow \bar{y} = \frac{128}{7}$	

Question			Generic Scheme	Illustrative Scheme	Max Mark
12.	(a)		$\bullet^1$ Create speed/distance triangle and annotate	$\bullet^1$ true speed = $\frac{1080}{2\cdot25}=480$  $\|\mathbf{v}_w\|^2=480^2+450^2-2\times480\times450\times\cos10°$	**3**
			$\bullet^2$ Calculate wind speed	$\bullet^2$ $86\cdot4$ km h^{-1}	
			$\bullet^3$ Calculate direction	$\bullet^3$ $\cos^{-1}\left(\frac{86\cdot4^2+450^2-480^2}{2\times86\cdot4\times450}\right)=105\cdot2°$ So bearing is $025\cdot2°$	

Notes:

1. Accept 025° or 205·2°
2. Any **angle** must include reference to a compass direction (from S25·2°W)

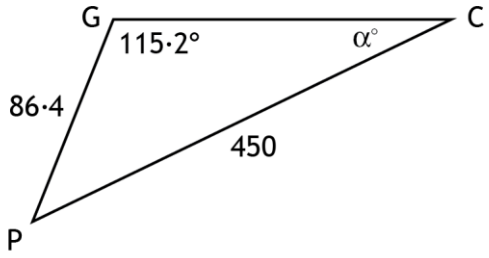
Commonly Observed Responses:

1. ***Direction*** of wind needed to be referenced.

	(b)	(i)	$\bullet^1$ State valid reason	$\bullet^1$ wind now acting against direction of travel	**1**

Notes:

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
		(ii)	•^{1} Redraw diagram (could be implied by mark 2)	•^{1} 	4
			•^{2} Calculate an angle and substitute	•^{2} $\frac{\sin 115\cdot 2}{450}=\frac{\sin C}{86\cdot 4}\Rightarrow C=10\cdot 0^\circ$ $P=54\cdot 8^\circ$	
			•^{3} Calculate true speed	•^{3} $v^2=450^2+86\cdot 4^2-2\times 450\times 86\cdot 4\times \cos 54\cdot 8^\circ$ true speed = $406\cdot 38$ km h^{-1}	
			•^{4} Calculate difference in travel time	•^{4} $t=\frac{1080}{406\cdot 38}=2\cdot 66$ hours = 2 hours 40 mins 25 mins longer	

Notes:

1. •^{4} Markers must work with candidate's appropriate rounding.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
Alternative solution by VECTORS					
12.	(a)		• ^{1} Create vector equation and use to find wind vector • ^{2} Calculate wind speed • ^{3}Calculate direction	•^{1} $2\frac{1}{4}\begin{pmatrix}450\cos 10°\\-450\sin 10°\end{pmatrix}+2\frac{1}{4}\mathbf{v}_{\mathrm{w}}=\begin{pmatrix}1080\\0\end{pmatrix}$ $\mathbf{v}_{\mathrm{w}}=\begin{pmatrix}36\cdot84\\78\cdot14\end{pmatrix}$ •^{2} $\lvert\mathbf{v}_{\mathrm{w}}\rvert=86\cdot4\,\mathrm{km\,h^{-1}}$ •^{3} $\tan^{-1}(\frac{78\cdot14}{36\cdot84})=64\cdot8°$ So wind bearing is 025·2°	**3**

Notes:

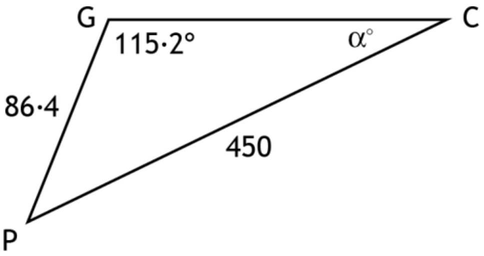
1. Accept 025°
2. Any angle must include reference to a compass direction (from S19°W)

Commonly Observed Responses:

	(b)	(i)	• ^{1} State valid reason	•^{1} wind now acting against direction of travel	**1**

Notes:

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
		(ii)	•¹ Redraw diagram (could be implied by mark 2)	•¹ 	4
			•² vector equation to find α°	•² $\begin{pmatrix}-450\cos\alpha \\ -450\sin\alpha\end{pmatrix}+\begin{pmatrix}86\cdot4\sin25\cdot2° \\ 86\cdot4\cos25\cdot2°\end{pmatrix}=\begin{pmatrix}-v \\ 0\end{pmatrix}$ $\alpha=10\cdot0°$	
			•³ Substitute in vector equation to find time of flight	•³ $t\begin{pmatrix}-450\cos10° \\ -450\sin10°\end{pmatrix}+t\begin{pmatrix}86\cdot4\sin25\cdot2° \\ 86\cdot4\cos25\cdot2°\end{pmatrix}=\begin{pmatrix}-1080 \\ 0\end{pmatrix}$ $t=2\cdot66$ hours = 2hr 40 mins	
			•⁴ Calculate difference in travel time	•⁴ Difference in time of flight 25 minutes	
Notes: 1. Markers must work with candidate's appropriate rounding.					
Commonly Observed Responses:					

Question			Generic Scheme	Illustrative Scheme	Max Mark
13.	(a)		$\bullet^1$ state integral to be used to find the volume with substitution	$\bullet^1$ $V=\int \pi y^2 dx$ $V=\int \pi e^{\frac{x}{6}} dx$	**3**
			$\bullet^2$ integrate and state limits	$\bullet^2$ $V=\int_{15}^{30} \pi e^{\frac{x}{6}} dx$ $=\left[6\pi e^{\frac{x}{6}}\right]_{15}^{30}$	
			$\bullet^3$ substitute in limits and calculate the volume	$\bullet^3$ $V=6\pi e^{5}-6\pi e^{2\cdot5}$ (2570cm^3)	
Notes: 1. $\bullet^3$ Accept any numeric answer correct to at least 3 significant figures (2570cm^3)					
Commonly Observed Responses:					
	(b)		$\bullet^4$ state volume equated to half original volume	$\bullet^4$ $6\pi e^{\frac{a}{6}}-6\pi e^{2\cdot5}=1285$	**3**
			$\bullet^5$ solve equation	$\bullet^5$ $e^{\frac{a}{6}}=80\cdot3$ $a=26\cdot3$	
			$\bullet^6$ interpret required solution	$\bullet^6$ $26\cdot3-15=11\cdot3\text{cm}$ Hence line should be positioned $10\cdot1\text{cm}$ up the side of the bowl.	
Notes:					
Commonly Observed Responses:					

Question			Generic Scheme	Illustrative Scheme	Max Mark
14.			$\bullet^1$ Consider force P parallel to the plane	$\bullet^1$ $P = W\sin\theta - \mu R_1$	**7**
			$\bullet^2$ Equilibrium perp to slope and substitution	$\bullet^2$ $R_1 = W\cos\theta$ $P = W\sin\theta - \mu\left(W\cos\theta\right)$	
			$\bullet^3$ Consider horizontal force Q along and perp to plane and equation for equilibrium.	$\bullet^3$ $Q\cos\theta + \mu R_2 = W\sin\theta$ $R_2 = W\cos\theta + Q\sin\theta$ $Q\cos\theta = W\sin\theta - \mu\left(W\cos\theta + Q\sin\theta\right)$	
			$\bullet^4$ Rearrange for μ	$\bullet^4$ $\mu = \dfrac{W\sin\theta - Q\cos\theta}{\mathrm{W}\cos\theta + Q\sin\theta}$ or $\mu = \dfrac{W\sin\theta - P}{W\cos\theta}$	
			$\bullet^5$ Equate expressions	$\bullet^5$ $\dfrac{W\sin\theta - Q\cos\theta}{\mathrm{W}\cos\theta + Q\sin\theta} = \dfrac{W\sin\theta - P}{W\cos\theta}$	
			$\bullet^6$ Simplify algebra	$\bullet^6$ $W^2\sin\theta\cos\theta - QW\cos^2\theta =$ $W^2\sin\theta\cos\theta - PW\cos\theta + QW\sin^2\theta - PQ\sin\theta$	
			$\bullet^7$ Use $\cos^2\theta + \sin^2\theta = 1$ to complete proof	$\bullet^7$ $\mathrm{P}(\mathrm{W}\cos\theta + Q\sin\theta) =$ $QW(\sin^2\theta + \cos^2\theta)$ $P = \dfrac{QW}{Q\sin\theta + \mathrm{W}\cos\theta}$	

Notes:

1. If candidates have block slipping up the slope, they cannot gain $\bullet^1$ but all other marks are available

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
15.	(a)		•^{1} Establish forces involved	•^{1} $ma = -6v - \frac{\lambda}{l}x$ or $-6v - 20x$	**2**
			•^{2} substitute values and re-arrange	•^{2} $0\cdot 25\frac{d^2x}{dt^2} = -6\frac{dx}{dt} - 20x$ $\frac{d^2x}{dt^2} + 24\frac{dx}{dt} + 80x = 0$	
Notes:					
Commonly Observed Responses:					
	(b)		•^{1} set up auxiliary equation	•^{1} $m^2 + 24m + 80 = 0$	**6**
			•^{2} solve quadratic equation	•^{2} $(m+20)(m+4) = 0 \Rightarrow m = -4$ or $m = -20$	
			•^{3} general solution	•^{3} $x = Ae^{-4t} + Be^{-20t}$	
			•^{4} initial condition $x = 0\cdot 2$ when $t = 0$	•^{4} $0\cdot 2 = Ae^0 + Be^0 \Rightarrow A + B = 0\cdot 2$	
			•^{5} differentiate to use initial condition for velocity	•^{5} $\frac{dx}{dt} = -4Ae^{-4t} - 20Be^{-20t} \Rightarrow -4A - 20B = 0$	
			•^{6} solve for *A* and *B*, particular solution	•^{6} $A = 0\cdot 25,\ B = -0\cdot 05$ $x = 0\cdot 25e^{-4t} - 0\cdot 05e^{-20t}$	
Notes:					
Commonly Observed Responses:					

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(c)		$\bullet^1$ differentiate to obtain an expression for acceleration	$\bullet^1$ $\ddot{x} = 4e^{-4t} - 20e^{-20t} = 0$	**3**
			$\bullet^2$ solve for t	$\bullet^2$ $e^{-4t} = 5e^{-20t}$ $-4t = \ln 5 - 20t$ $t = \frac{1}{16}\ln 5$	
			$\bullet^3$ substitute for t to give displacement, x	$\bullet^3$ $x = \frac{1}{4}e^{-0\cdot25\ln 5} - \frac{1}{20}e^{-1\cdot25\ln 5} = 0\cdot160\,\text{m}$	
Notes:					
Commonly Observed Responses:					
16.	(a)		$\bullet^1$ state the horizontal and vertical equations of motion	$\bullet^1$ $x = Vt\cos\theta \quad y = Vt\sin\theta - \frac{1}{2}gt^2$	**3**
			$\bullet^2$ rearrange for t and start substitution	$\bullet^2$ $t = \frac{x}{V\cos\theta}$ $y = \left(V\sin\theta \times \frac{x}{V\cos\theta}\right) - \frac{1}{2}g\left(\frac{x}{V\cos\theta}\right)^2$	
			$\bullet^3$ obtain required equation	$\bullet^3$ $y = x\tan\theta - \frac{gx^2}{2V^2\cos^2\theta}$ $y = x\tan\theta - \frac{gx^2}{2V^2}\left(1 + \tan^2\theta\right)$	
Notes: 1. **$\bullet^1$ can be implied by $\bullet^2$ and $\bullet^2$ can be implied by $\bullet^3$**					
Commonly Observed Responses:					

Question			Generic Scheme	Illustrative Scheme	Max Mark
Alternative solution for •^{1} using calculus					
			• ^{1} state horizontal and vertical equations of motion using calculus	$\frac{d^2x}{dt^2}=0$ $\frac{dx}{dt}=c$ when $t=0$ $\frac{dx}{dt}=V\cos\theta$ $c=V\cos\theta$ $\mathrm{x}=\int\left(v\cos\theta\right)dt=\left(V\cos\theta\right)t+c_2$ when $t=0$ $x=0$ $c_2=0$ $x=\left(V\cos\theta\right)t$ $\frac{d^2y}{dt^2}=-g$ $\frac{dx}{dt}=\int\left(V\sin\theta\right)dt=-gt+c_3$ when $t=0\frac{dy}{dt}=V\sin\theta c_3=V\sin\theta$ $y=\int\left(V\sin\theta-gt\right)dt$ $y=\left(V\sin\theta\right)t-\frac{1}{2}gt^2+c_4$ when $t=0$ $y=0$ $c_4=0$ $y=\left(V\sin\theta\right)t-\frac{1}{2}gt^2$	**3**
Alternative solution for •^{1} using equations of motion					
			• ^{1} state horizontal and vertical equations of motion using equations of motion	$x=V\cos\theta\times t$ $s=ut+\frac{1}{2}at^2$ $y=\left(V\sin\theta\right)t-\frac{1}{2}gt^2$	**3**

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		$\bullet^4$ Use equation from part (a) to obtain two equations for h	$\bullet^4$ $h = 4h\tan\theta - \frac{16h^2 g}{2V^2}\left(1+\tan^2\theta\right)$ $h = 5h\tan\theta - \frac{25h^2 g}{2V^2}\left(1+\tan^2\theta\right)$	**5**
			$\bullet^5$ equate expressions and some algebraic manipulation	$\bullet^5$ $\frac{h^2 g}{V^2}\left(1+\tan^2\theta\right) = \frac{2}{25}\left(5h\tan\theta - h\right)$ $\frac{h^2 g}{V^2}\left(1+\tan^2\theta\right) = \frac{1}{8}\left(4h\tan\theta - h\right)$	
			$\bullet^6$ Eliminate V	$\bullet^6$ $\frac{2}{25}(5h\tan\theta - h) = \frac{1}{8}(4h\tan\theta - h)$	
			$\bullet^7$ simplify and eliminate h	$\bullet^7$ $80h\tan\theta - 16h = 100h\tan\theta - 25h$ $20\tan\theta = 9$	
			$\bullet^8$ find angle of projection	$\bullet^8$ $\tan\theta = \frac{9}{20} \quad \Rightarrow \theta = 24\cdot2^\circ\ (0\cdot423)$	
Notes:					
Commonly Observed Responses:					

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		**Alternative solution**		
			$\bullet^{4}$ Express as a quadratic and obtain equation.	$\bullet^{4}$ quadratic passes through the origin so is of the form $y = ax^2 + bx$ Passes through the points $(4h, h)$ and $(5h, h)$ $h = 16h^2a + 4hb$ $h = 25h^2a + 5hb$	
			$\bullet^{5}$ solve simultaneous equations to obtain expressions for a and b	$\bullet^{5}$ $5h = 80h^2a + 20hb$ $4h = 100h^2a + 20hb$ $a = \frac{-1}{20h}$ and $b = \frac{9}{20}$	
			$\bullet^{6}$ state equation of the trajectory	$\bullet^{6}$ $y = \frac{-1}{20h}x^2 + \frac{9}{20}x$	
			$\bullet^{7}$ determine gradient when $x = 0$	$\bullet^{7}$ $\frac{dy}{dx} = \frac{-1}{10h}x + \frac{9}{20}$ at $x = 0$ $\frac{dy}{dx} = \frac{9}{20}$	
			$\bullet^{8}$ solve to find angle of projection	$\bullet^{8}$ $m = \tan\theta \Rightarrow \tan\theta = \frac{9}{20}$ $\theta = 24\cdot2^{\circ}$	
Notes:					
Commonly Observed Responses:					

Question			Generic Scheme	Illustrative Scheme	Max Mark
17.	(a)	(i)	•¹ Consider energy at two positions	•¹ at starting position $E_k = \frac{1}{2}mu^2$ Elsewhere $E_k + E_p = \frac{1}{2}mv^2 + mg\left(r - r\cos\theta\right)$	**4**
			•² Use conservation of energy	•² $\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg\left(r - r\cos\theta\right)$	
			•³ Find expression for velocity	•³ $v^2 = u^2 - 2rg\left(1 - \cos\theta\right)$	
		(ii)	•⁴ $F = ma$ radially and show expression for tension	•⁴ $T - mg\cos\theta = \frac{mv^2}{r}$ and complete	**2**
			•⁵ Interpret condition for full circles	•⁵ $T > 0$ when $\theta = 180°$ and substitute	
			•⁶ Find expression for u	•⁶ $u > \sqrt{5rg}$	
Notes: 1. accept $T \geq 0$ and $u \geq \sqrt{5rg}$					
Commonly Observed Responses: Condition for complete circle given as $v > 0$					
	(b)		•⁷ Interpret condition for string going slack and substitute	•⁷ $T = 0$ and substitute	**3**
			•⁸ Find angle	•⁸ $\cos\theta = -\frac{2}{3}$	
			•⁹ Find height	•⁹ $h = r - r\cos\theta = \frac{5}{3}r$ $h = r - r\cos\theta = \frac{5}{3}r$	

[END OF MARKING INSTRUCTIONS]